



Is the transfer function method reliable in a European building context? A theoretical analysis and a case study in the south of Italy

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Abstract

The available tools for dynamic simulation of the buildings thermal behaviour are manifold, and the most modern ones, known as TRNSYS, are founded upon the use of the Z-transform (ZT) set also called transfer function method (TFM). The transfer function method (TFM), recommended by American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE), is one of the most modern tools available to solve heat transfer problems in building envelopes and environments. TFM utilises Z-transforms to solve the equation system that describes the heat transfer in a multi-layered wall. Due to an analogy with an electric circuit, it is possible to write the equation system in a matrix suitable to be solved by computer.

The authors have investigated TFM mathematical features, especially concerning the reliability and the quality of the thermal dynamic simulations. Using some basilar control systems tools such as plots, step response and model validation, results show clearly that, for a massive building, a simple application of TFM very often fails.

The target of the work was to identify an automatic procedure able to optimise the reliability of the simulation by selecting and using in a more correct way poles and the residuals of the transfer functions.

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1. Introduction—state of Art

Analysis of the thermal performance of buildings during design allows the developer to choose the best materials for the local climatic characteristics. Such a preliminary analysis is very important because it allows energy saving and a correct bio-climatic approach.

This analysis can be done with simulation programs that are able to analyse the heat transfer phenomena. Usually dynamic modelling is required and one-dimensional heat transfer is normally assumed. Commonly used techniques are finite difference methods (ESP, APACHE-sim, K2, SERI-RES, HTB-2), convolution-based *conduction transfer function* methods (BLAST, TARP, DOE-EnergyPlus, TRNSYS) [1,2].

A large part of the currently employed models are still deterministic. Their purpose is to predict the time evolution of the building plant system behaviour from a given set of driving forces, expressed as explicit time functions. Taking into account that the deterministic approach often fails due to the high degree of randomness in real systems, developers have to cope with this new challenge: to face the uncertainty. Furthermore, new techniques in building and HVAC system modelling such as neural networks and genetic algorithms can contribute to overcome the linearity assumption [3,4].

Dynamics are modelled using a discretisation of the time. A sampling time-step is normally used, the main determinant of which is the time-resolution of available weather data (usually 1 h). Much non-linearity occurs in the equations representing the thermal balance of the room. However in the type of the plant model usually adopted, the control characteristics take the form of linear functions relating room heating or cooling inputs to zone temperatures, which can readily be incorporated into the linear equation sets representing the room's thermal balance. Solution techniques vary between programs. All, however, employ some combination of linearisation, matrix manipulation and iterative methods.

Research-oriented building simulation programs are created for a variety of purposes. Being designed for use by researchers with specialised knowledge, they tend to be less user-friendly than their commercial counterparts [2].

However, this has not prevented some of them—ESP-r, HTB-2, SERI-RES and TRNSYS, for example—from being used, from time to time, in commercial projects.

The advantage of the transfer function method (TFM) is the speed in the case of application to a single multi-layered wall. But, when the TFM calculation methodology is applied to a thermal zone, the manual calculation becomes impossible. In the simplest case one should have to calculate the determinant of a seventh-order matrix several thousands times in order to calculate only few poles. Also knowing the coefficients of the transfer functions, the method should have to be applied tens of times for different input–output couples. In the case of thermal zones simulated with the TFM the advantage is not in the speed or the manual calculation (virtually impossible). The advantage is that it is easy to arrange a multi zone model taking advantage of the linearity of the algorithm. Furthermore, since the model is flexible in defining the boundary conditions, it can

be coupled with other tools, without reducing the accuracy of the whole model [5]. The disadvantage is the loss of information due to the linearity assumptions, but previous studies have estimated that the linearity assumptions in TFM, for most building materials and operation strategies imply no significant difference between the results of this model and those of other numerical models [5].

Although the scientific community has been involved in this field for more than 30 years, most of the available commercial software, which are largely diffused and accepted all over the world, produces some problems when utilised to simulate massive buildings.

In fact, it is also known how the most diffused commercial software for the non-steady-state thermal simulation does not assure the exact determination of the transfer function coefficients for massive building structures. Furthermore, most of this software does not permit changes and to fit basic parameters of the calculus.

The authors have developed THELDA 2000, a software able to determine the possible sets of coefficients of the transfer functions [6–9]. The method developed allows the selection of the number of transfer function coefficients, number of poles and the sampling period. A thermal zone from an ancient very massive building located in Marsala (Sicily) has been simulated using THELDA 2000. Thermo-physical features of the walls and weather data from Italian standards have been used.

2. The software THELDA 2000

Thermal elements dynamic analysis (THELDA 2000) is able to simulate the thermal behaviour of a single thermal zone using the Z-Transform and the TFM [9].

THELDA 2000 is also capable of changing the sampling period, and to fix the preferred number of poles and number of coefficients used for the simulation. Besides, for appraising the precision of the calculation, THELDA 2000 performs simulations even with a different mathematical approach like Fourier series expansion.

A remarkable and innovative characteristic of this software is the possibility to modify, through a user-friendly interface, any parameters linked to the application of TFM changing, by this way, the accuracy of the analysis. This feature is rarely available in other common tools for thermal building simulation and it gives the software characteristics of flexibility and innovation with respect to the software programs mostly used in this field.

We have also implemented an innovative algorithm that permits to reorder the residuals for every transfer function. Results show clearly that the possibility to reorder residuals has a strong influence on the reliability of the simulations carried out.

Concerning the response of the system, it represents a matrix in which the first six columns are the hourly behaviour of the temperature on the inner surface of each wall and the last column represents the hourly temperature of the air (free floating simulations). For the investigated case study the input signal is the air temperature and solar radiation on the external surface of the first wall calculated for the city of Marsala in the south of Italy in July (orientation south-east). Moreover, other input signals are radiative heat flux through a window and the ventilation air. The input signals are periodic and repeat themselves every 24 h.

3. Thermal balance of the thermal zone

On the external surface of the wall bordering with the outdoor the thermal balance is simplified by using the concept of air–sol temperature. The inner surface of the wall presents a heat flux due to the convective exchange with the internal air and due to radiative exchange with other surfaces. It also receives a thermal flux from the internal environment due to electric devices or cooling plants. The thermal exchange between the glass and the internal air is summarized by an exchange coefficient. Solar radiation is distributed by means of a simple linear model related to the extension of the inner surfaces.

In order to evaluate the accuracy of calculations carried out by using a given set of ZT coefficients, we have to compare them with a reference response coming from a procedure having a different mathematical background and even able to give the time–continuous response of the system.

Thus, in order to evaluate the reference response if we want to control the reliability of the coefficient sets evaluated by means of diverse input signals we have to:

- Use input signals with the typical time variation of the physical phenomena and then avoiding steps, ramps, etc.
- Adopt an evaluating approach which differs from that employed to calculate ZT coefficient sets, and then exclude Laplace Transform (LT).

We carried out a comparison between simulation data obtained from a Fourier steady-state algorithm and those obtained from the TFM [10]. To perform this comparison it has been necessary to define, for any generic input signal, a rule of interpolation able to reproduce a realistic behaviour starting from the sampled data.

So, the input signal has been represented by the function $X(t)$, which is not discrete within the generic interval of the sample t_n, t_{n+1} defined by the expression:

$$X(t) = at^3 + bt^2 + ct + d \quad (1)$$

in which the coefficients a, b, c, d are calculated imposing the following constraints:

- (1) the $X(t)$ assumes the values $X(t_n)$ and $X(t_{n+1})$,
- (2) the tangent to the curve at the point $[t_n, X(t_n)]$, forms equal angles with the segments joining $X(t)$ to $X(t_{n-1})$ and $X(t_n)$ to $X(t_{n+1})$.

In periodic state any wave $f(t)$ with period T can be analysed into the sum of sinusoidal waves of different frequencies:

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)] \quad (2)$$

Supposing that $f(t)$ is known in correspondence to the N instants in which the period T was subdivided, coefficients are:

$$\begin{cases} a_0 = \frac{2}{N} \sum_{k=0}^{N-1} f\left(\frac{kt}{N}\right) \\ a_n = \frac{2}{N} \sum_{k=0}^{N-1} f\left(\frac{kt}{N}\right) \cdot \cos\left(\frac{2\pi kn}{N}\right) \\ b_n = \frac{2}{N} \sum_{k=0}^{N-1} f\left(\frac{kt}{N}\right) \cdot \sin\left(\frac{2\pi kn}{N}\right) \end{cases} \quad (3)$$

Then, calculating the time response to each harmonic component and recombining all of them, one can obtain the system response to the generic $f(t)$.

The calculated temperatures have been compared to those obtained by Fourier's analysis in periodic steady-state conditions.

We based the comparison on the percentage mean error parameter PME:

$$\text{PME}_k = \frac{1}{24} \sum_{\tau=1}^{24} \frac{|T_{Zk}(\tau) - T_{Fk}(\tau)|}{T_{Fk}(\tau)} \cdot 100 \quad (4)$$

$$\text{PME}_{\text{air}} = \frac{1}{24} \sum_{\tau=1}^{24} \frac{|T_{Z\text{air}}(\tau) - T_{F\text{air}}(\tau)|}{T_{F\text{air}}(\tau)} \cdot 100 \quad (5)$$

where T_Z and T_F are the hourly temperatures at generic time τ for the k th inner surfaces or for the air, calculated using the ZT and the Fourier analysis respectively. To exit from transient response, the calculation with ZT is repeated until the signal is stabilised [11].

4. Analysis of the case study

We have observed that there are at least three parameters influencing the reliability of the simulation:

1. The sampling period Δ .
2. The number of poles n_p .
3. The number of coefficients n_c .

In order to select the *best model* representing the real system we carry out an analysis of the transfer functions obtained. For further details on the TFM method see the [Appendix A](#).

Tools that we are using for analysis are based on some control systems concepts [12–14].

We report some definitions:

Non-minimal phase zeros: In the z -domain, a zero z_i is said to be a *non-minimal phase* zero if it is outside the unit circle: $|Z_i| > 1$.

Stable poles: In the z -domain a pole p_i is said to be stable if it lies inside the unit circle: $|p_i| < 1$

Step response: The response to a step signal is used in control literature to test the steady-state condition.

Frequency response: The response of a linear system to a sinusoidal input is referred to as the system's *frequency response*. For continuous-time systems, a stable linear system with transfer function in the Laplace domain $G(s)$, if excited by a sinusoid, will exhibit a sinusoidal output with the same frequency of the input. The magnitude, $A(\omega)$ of the output with respect to the input is $|G(j\omega)|$ and the phase $P(\omega)$ is $\angle G(j\omega)$, letting s take values along the imaginary $j\omega$ -axis.

It is possible to define the frequency response also for discrete-time systems. If the system has transfer function $G(z)$, we define its magnitude and phase for z taking on values around the unit circle by $G(e^{j\omega T}) = A(\omega T)e^{jP(\omega T)}$. If an amplitude one sinusoid is applied, then in the steady state, the response samples will be on a sinusoid of the same frequency ω_0 with amplitude $A(\omega_0 T)$ and phase $P(\omega_0 T)$.

Bode plots: Plotting for each frequency the *magnitude* in dB and the *phase* of the output response of each sinusoid input on a semi-logarithmic scale we get the Bode plots.

Authors [15,16] have utilised THELDA 2000 to perform a simulation of a single thermal zone belonging to a massive ancient building in the city of Marsala. The features of this building give some difficulties when carried out by thermal simulation using other software; so we have decided to perform a deep analysis over the transfer functions of this building.

The room used for the analysis is composed of six walls, one bordering the external environment. Every wall is composed of a sandstone layer in the middle and inside–outside plaster. We have analysed a room where all walls have identical characteristics. In order to study the reliability of the TFM related to the thermal inertia we have considered several cases: the thickness of the sandstone layer varies from 0.4 to 0.8 m. Such features are typical of the Mediterranean building historical heritage.

To evaluate the reliability of the transfer functions by using system control tools we used an approach based on the method described in [13,14].

We have built the transfer function $G(z)$ that links the external temperature with the inner temperature of the room taking an increasing number of poles and the corresponding residuals. No optimal selection of the poles and residual has been carried out up to now.

Let $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ be the corresponding transfer function of order $n_p = 5$, $n_p = 10$ and respectively, $n_p = 15$.

In Figs. 1–3 the output response of $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ to the external temperature step signal $u(t) = 50^\circ$ is shown for different sampling period ($\Delta = 1, 2$ and 3 h).

It is evident that the model obtained for $\Delta = 1$ h which output response is shown in Fig. 1 is not correct. After the transient the output does not reach the steady-state value of 50° . In particular, increasing the number of poles leads to worse behaviour.

Increasing the sampling period, $\Delta = 2$ h, we see in Fig. 2 that for $n_p = 5, 10$ the steady-state behaviour is better, even if still for $n_p = 15$ the steady-state output is not correct.

Finally, for $\Delta = 3$ h, as shown in Fig. 3, the steady-state behaviour is the one expected. We point out that increasing the number of poles and consequently the precision level, instead of improving the quality of physical response, leads to a drastic worsening.

Looking at the transient response in Figs. 2 and 3, the presence of a non-minimal phase system is also evident. Indeed, a transfer function with unstable zeros (non-minimal phase) presents a step response that, at the very beginning, has a slope that is opposite to the sign of the step input: for positive step input the output is initially decreasing.

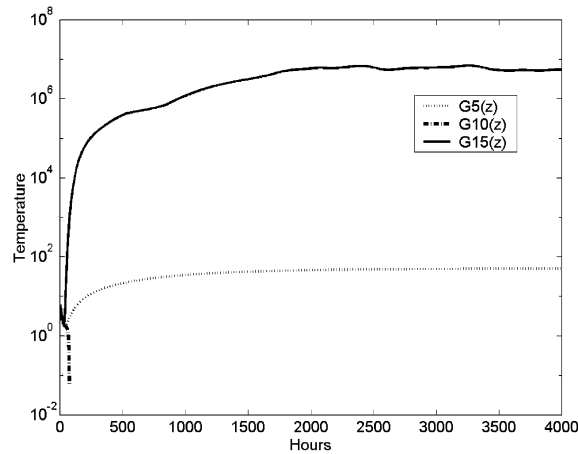


Fig. 1. Step response output for the transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ with sampling period $\Delta = 1$ h.

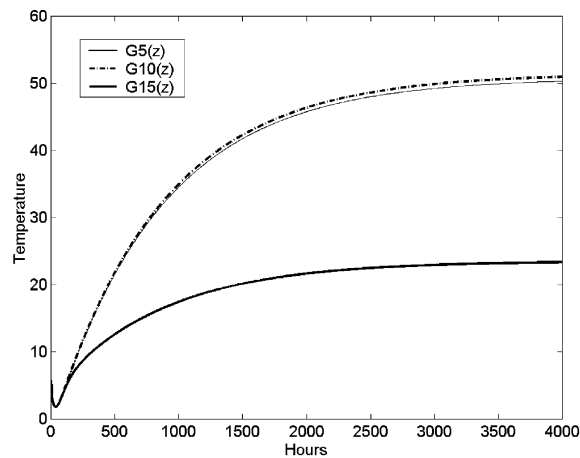


Fig. 2. Step response output for the transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ with sampling period $\Delta = 2$ h.

In Figs. 4–6 we have reported the discrete-time Bode plots of $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ for sampling time $\Delta = 1, 2$ and 3 h. Looking at Figs. 4–6 it is evident that when the sampling period is big enough, the Bode plots of the different transfer functions are identical for low frequencies, but on increasing the number of poles, the frequency response is slightly different at high frequency.

From Fig. 4 it is clear that $G_{10}(z)$ and $G_{15}(z)$ are not correct for low frequencies (implying that the steady-state value in the step response is completely wrong). In fact, the response signal of our system must always be zero or less: a thermal system can not amplify the input signal so we should obtain bode plots always in the negative part of the plane. A magnitude greater than zero means an amplification of the output signal.

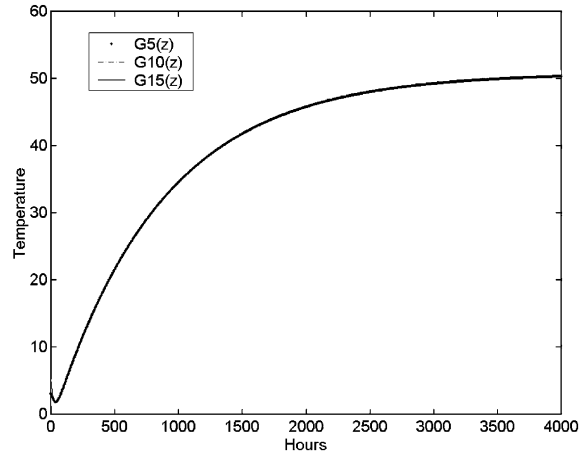


Fig. 3. Step response output for the transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ with sampling period $\Delta = 3$ h.

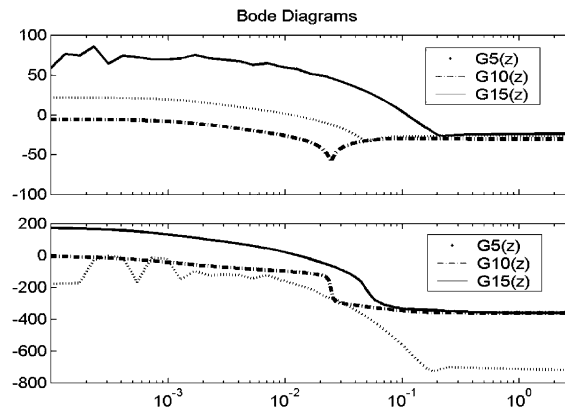


Fig. 4. Bode plots for the transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ with sampling period $\Delta = 1$ h. Amplitude on the top and phase on the bottom.

Comparing Figs. 4–6 it is manifest that with a sampling period $\Delta = 1$ h for $n_p = 15$ at low frequency the magnitude is not correct (a magnitude greater than 50 dB means a steady-state gain greater than 300 compared with the correct one!).

We have plotted the poles of the discrete-time transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ in the complex plane and, from their locations we checked that the system is stable. In particular, from the physical interpretation of the system we know that the poles must be all real. This is true for $\Delta = 2, 3$ but not for $\Delta = 1$, when the model obtained is not correct due to numerical problems (some poles are complex conjugate).

Furthermore we have plotted the zeros of the discrete-time transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ in the complex plane. We checked that the system is characterised by a lot of non-minimum phase zeros especially for $\Delta = 1$ and for $G_{10}(z)$ and $G_{15}(z)$.

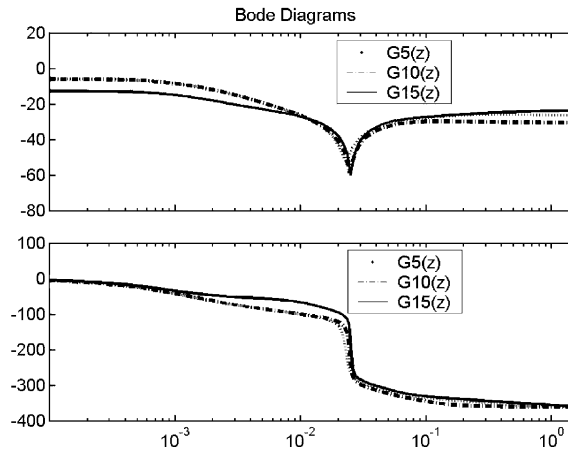


Fig. 5. Bode plots for the transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ with sampling period $\Delta = 2$ h.

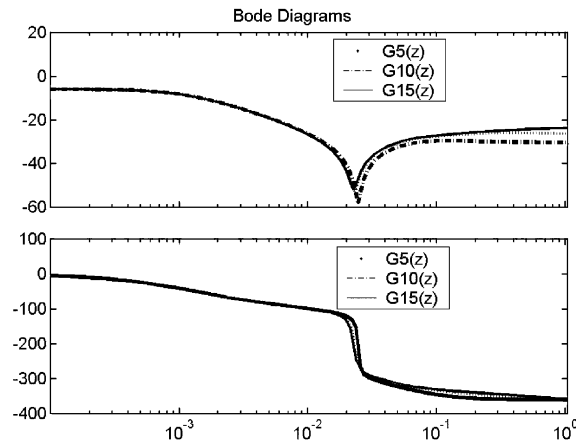


Fig. 6. Bode plots for the transfer functions $G_5(z)$, $G_{10}(z)$ and $G_{15}(z)$ with sampling period $\Delta = 3$ h.

Based on the previous analysis of the poles and zeros location, bode diagram and step response, we can set the following possible recipes in order to overcome numerical problems and set up model selection rules.

1. *The best model is not always the biggest in terms of number of poles.*
2. In our example, a model with only five poles ($n_p = 5$) can represent the system as well as the one with 15 poles, without any numerical problem. Only at high frequencies the behaviour is more approximated.
3. *Increasing the sampling period is always a good way to eliminate numerical problems.*

In the example reported, and in many other simulation examples we have collected, increasing the sampling time of (here with $\Delta = 3$ h) all the possible models and they are good in terms of

performance: numerical problems are avoided. On the other hand, a simulation model with sampling period equal to 3 h is not very useful due to the lack of information.

As shown in literature, *the best* model is not always *the biggest* [10]. The choice of optimal sampling period has dramatically improved the model for the case study under consideration, which is not tractable with available commercial software.

5. Choice of significant poles: identification of a procedure for the optimal selection of poles and residuals

Results from the analysis developed on building types characterised by high values of thermal inertia, show that the optimal choice of initial parameters of calculation is very important. We have found that these building structures need particular attention in choosing the number of poles and sampling period to be used in the Z-transform method [17,18].

We have identified an automatic procedure able to optimise the reliability of the simulation. Applying to the TFM some concepts coming from system theory, it is possible to obtain a drastic improvement in the quality of the simulation. Our approach is based on the following method, named Procedure I.

Given a Laplace transfer function

$$G(s) = \sum_{n=1}^N \frac{d_n}{(s - p_n)} \quad (6)$$

Let $\hat{d}_n$ be the ordered residuals such that $|\hat{d}_{i-1}| > |\hat{d}_i| > |\hat{d}_{i+1}|$ whose corresponding poles are $\hat{p}_i$. Then

$$G(s) = \sum_{n=1}^N \frac{d_n}{(s - p_n)} = \sum_{n=1}^N \frac{\hat{d}_n}{(s - \hat{p}_n)} \quad (7)$$

Let us consider the truncated

$$\hat{G}(s) = \sum_{n=1}^{\hat{N} < N} \frac{\hat{d}_n}{(s - \hat{p}_n)} \quad (8)$$

This is obtained by getting rid of the non-significant poles corresponding to very low values of the residuals and taking into account only the $\hat{N}$ residuals (and corresponding poles) such that $|\hat{d}_i| > \sigma$, where σ is a prefixed value. It is well known from any classical control textbook that only the significant poles add information to the overall system. We have observed that it is possible to eliminate the residuals and the corresponding poles when residuals present values that have a value lower than $\sigma = 10^{-10}$.

Applying the new method for the choice of the only significant poles the quality of the simulation has a drastic improvement. In the following Figs. 7–12 we show data from simulations carried out using two different procedures:

Procedure I utilises the new algorithm, it reorders the residuals cutting off values lower than 10^{-10} , consequently eliminating the corresponding poles.

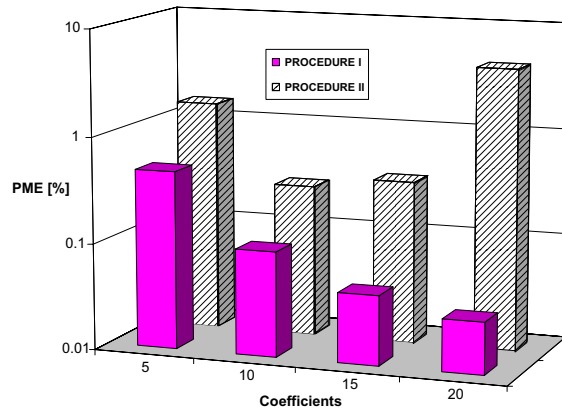


Fig. 7. PME for the surface bordering with the external environment; comparison between Procedure I and Procedure II; thickness of walls = 0.4 m. Sampling period = 1 h.

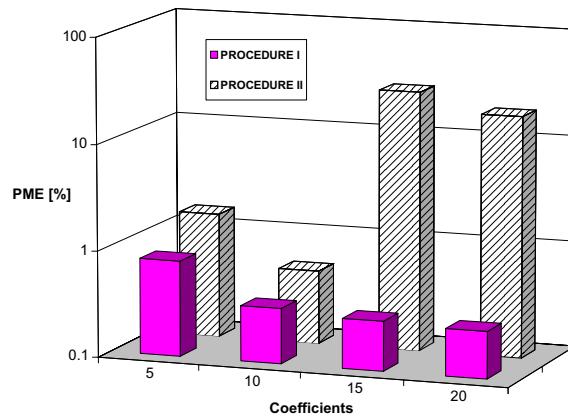


Fig. 8. PME for the surface bordering with the external environment; comparison between Procedure I and Procedure II; thickness of walls = 0.6 m. Sampling period = 1 h.

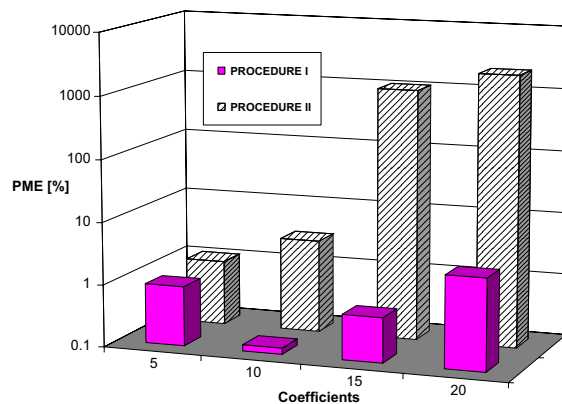


Fig. 9. PME for the surface bordering with the external environment; comparison between Procedure I and Procedure II; thickness of walls = 0.8 m. Sampling period = 1 h.

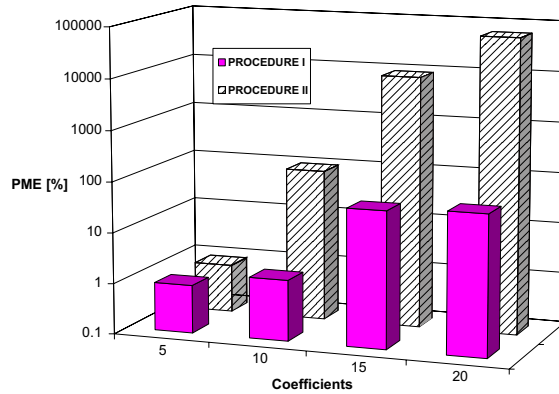


Fig. 10. PME for the surface bordering with the external environment; comparison between Procedure I and Procedure II; thickness of walls = 1 m. Sampling period = 1 h.

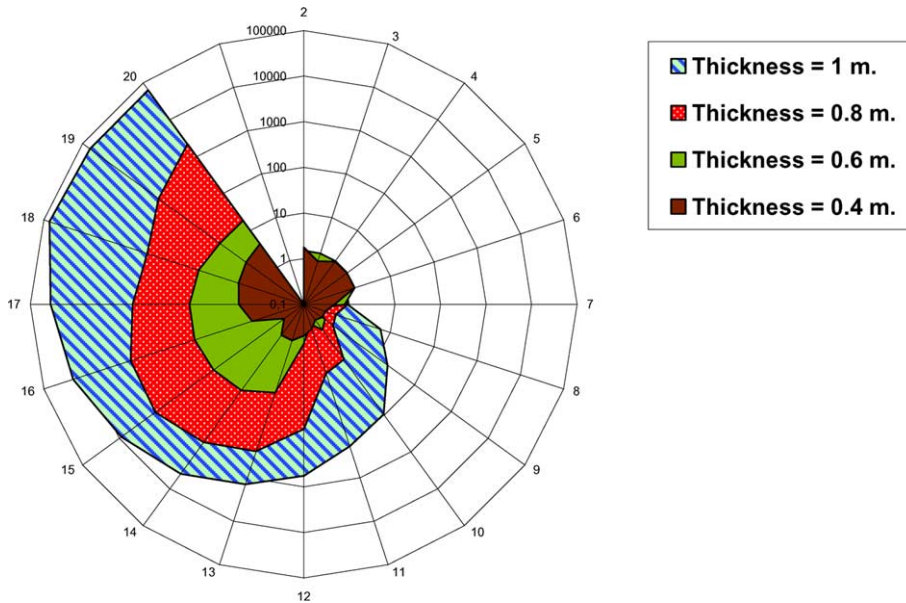


Fig. 11. PME profile varying the walls thickness and the number of employed poles. No optimal choice of poles and residuals (procedure II), sampling period = 1 h.

Procedure II is the normal procedure utilised by the TFM and no optimal selection of residuals and poles is performed. Once the Procedure I has selected a $\hat{G}(s)$ with only significant poles, the $\hat{G}_5(z)$, $\hat{G}_{10}(z)$, $\hat{G}_{15}(z)$ and $\hat{G}_{20}(z)$ have been evaluated.

In Procedure II starting from $G(s)$ with no reordering and truncation, we evaluate $G_5(z)$, $G_{10}(z)$, $G_{15}(z)$ and $G_{20}(z)$.

Figs. 7–9 show that the simulations carried out using the procedure I have a very good performance in terms of PME. It is evident that models obtained for $\Delta = 1$ h and using procedure II are

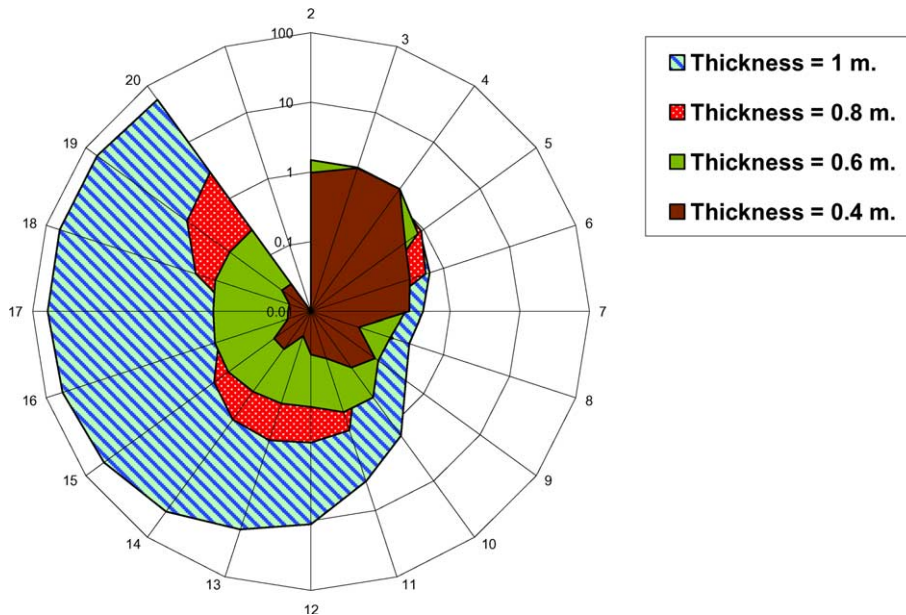


Fig. 12. PME profile varying the walls thickness and the number of employed poles. Optimal choice of poles and residuals (procedure I), sampling period = 1 h.

not correct because the PME is very high. Besides increasing the number of poles the behaviour is getting worse. On the other hand the same models using the procedure I give good results with PME lower than 1%.

6. Conclusions

The widespread mathematical models adopted for the simulation of non-steady-state thermal behaviour of buildings, if applied just as they are in a Mediterranean context, due to the particular thermophysical, typological, structural and technological characteristics of this geographical area, could give rise to numerical problems. In particular, the high thermal inertia of the buildings belonging to the historical centres of Southern European cities, along with the intrinsic characteristics of the materials employed, the arrangement of building types and the particular urban layout, do not enable simulations to be carried out with the required degree of accuracy. Thus, even if there are simulation models widely used and appreciated all over the world, their application to the buildings in our historical European town centres is still a doubtful matter.

With the present analysis, some solutions to the main numerical problems connected to the use of a standard software package for massive buildings have been shown. In particular, according to the authors' previous papers [10,15,16], it is evident that model validation and selection of the parameters is crucial. The two main points of [10] were the following:

1. The best model is not always the largest in terms of number of poles.
2. Increasing the sampling period is a good way to eliminate malfunctioning.

3. Furthermore, the procedure of poles and residuals optimal selection gives a dramatic improvement to the quality of simulations.

In the present work an efficient way to choose the only significant poles influencing the dynamics and the behaviour of the model has been presented. Combining this new procedure that picks the optimal poles and increasing the sampling period, all the numerical problems disappeared. Clearly, the “best” model is the one giving raise to the lowest PME. For example, considering a thickness of the walls of 0.8 m (as reported in Fig. 9), all the models obtained with the new procedure, even for $\Delta = 1$ h, presented a PME less than 1%, but the best model is $\hat{G}_{10}(z)$ showing a PME = 0.12%.

Calculation models largely diffused (such as DOE, TRNSYS) well suited for structures exhibiting low thermal inertia and capable of singling out the radiative part of thermal exchanges, could be hardly adapted to a European context. The European rules, in the present state-of-art, recognise these limits and uncertainties: in details, the Technical Committee TC98 (as part of the European Committee for Standardisation (CEN)), is still debating the accuracy level and the applicability of either a dynamic simulation carried out using detailed models, or a steady-state model dynamically corrected taking into account the effect of buildings thermal mass.

Our research makes clear, in fact, the limits of the most popular simulation models in describing, with suitable accuracy and usefulness, building structures exhibiting high thermal inertia.

DOE and TRNSYS, two of the most diffuse software for thermal building simulation, use the ASHRAE TFM calculation methodology. Currently the attention is on integrated software where the user can appreciate different aspects such as visual, acoustic and thermal comfort, life cycle assessment, etc., coming from various project choices. Referring to the quality of a thermal dynamic simulation there are some limits, particularly for TRNSYS, in the case of buildings characterised by the high thermal inertia, typical of traditional European building. Within this framework we think our work highly relevant because we point out the critical points in the quality of a thermal simulation performed by ASHRAE TFM calculation methodology.

Appendix A

The thermal behaviour of the zone is described by a system's equations that take into account the thermal balance of inner air and of the energy exchanges between the inner air and the internal surfaces delimiting the zone. Using the Laplace transform it is possible to describe the heat transmission within the walls by means of the transmission matrix of the single layers composing the walls [19]. Finally it is possible to obtain the following linear system's equations:

$$(M) \times (V) = (U) \quad (\text{A.1})$$

where the matrix of only the coefficients of the unknowns is (M) , (V) is the matrix of the unknown and (U) is the matrix of the known terms.

To find the transfer function $G(s)$ (s is the Laplace complex variable) in the Laplace domain it is useful that the system is solicited by a unitary impulse input. In these conditions the output of the system coincides with the transfer function.

Thanks to the linearity of the system the generic output can be obtained by using superimposition of the effects as sum of the partial outputs induced by single input signals. By this way for each output can be defined as many partial transfer functions as the inputs of the system. These partial transfer functions can be identified determining the generic output of the system imposing that:

- All of the terms composing the coefficients of the matrix (U) have to be zero except the term related to the input signal to which the partial transfer function is related to.
- Furthermore, the input signals (such as the external temperature, the solar radiation on the inner surfaces, etc.) to which the partial transfer function is referred, has to be unitary.

Referring to the above system's equations, the transfer function that links the generic temperature T_x to the generic input signal can be obtained solving the system's equation with the Cramer method:

$$G_{T_x, I_i}(s) = \frac{\det[M(T_x)]}{\det[M]} \quad (\text{A.2})$$

where $\det[M(T_x)]$ is the determinant of the matrix (M) in which we have replaced the column related to the unknown T_x with the coefficients of the matrix (U) in which was deleted all the terms not referred to the x input. At the same time, all the terms of the matrix (U) linked to the x input are unitary. Assuming that even in this case, to obtain the ZT coefficients, we are employing linear ramp input, then the generic partial output caused by the unitary input will be:

$$T_{x,i}(s) = \frac{1}{s^2} \frac{\det[M(T_x)]}{\det[M]} = \frac{1}{s^2} \frac{\text{NUM}(s)}{\text{DEN}(s)} = \frac{C_0}{s^2} + \frac{C_1}{s} + \sum_{n=1}^{\infty} \frac{\text{res}_n}{s - s_n} \quad (\text{A.3})$$

in which the poles s_n are obtained individuating the values of s that make null the determinant $\det[M]$. Also in this case it is useful to make the substitution $s = j\delta$ that makes us able to search poles only in the real numbers domain. Once obtained the generic S_n , we can calculate with automatic procedure:

$$\text{res}_n = \left[\frac{\text{NUM}(s)}{s^2 \text{DEN}'(s)} \right] \Big|_{s=s_n} = \frac{\det[M(T_x)]|_{s=s_n}}{s_n^2 \frac{d}{ds} \{\det[M]\}|_{s=s_n}} \quad (\text{A.4})$$

Analogously, it is possible to calculate C_0 and C_1 . Once obtained these coefficients we can calculate the ZT of the generic partial output:

$$T_{x,i}(z) = \frac{\text{num}(z)}{\text{den}(z)} I(z) = \frac{\text{num}_0 + \text{num}_1 z^{-1} + \text{num}_2 z^{-2} + \dots + \text{num}_n z^{-n}}{1 + \text{den}_1 z^{-1} + \text{den}_2 z^{-2} + \dots + \text{den}_n z^{-n}} I(z) \quad (\text{A.5})$$

in which

$$\frac{\text{num}(z)}{\text{den}(z)} = \frac{\frac{C_0 \Delta}{(1 - z^{-1})^2} + \frac{C_1}{(1 - z^{-1})} + \sum_{n=1}^{\infty} \frac{\text{res}_n}{1 - e^{s_n \Delta} z^{-1}}}{\frac{C_0 \Delta}{z(1 - z^{-1})}} \quad (\text{A.6})$$

Applying the definition of TFM and expanding the terms we can write that:

$$\begin{aligned} T_{x,i}(0) + T_{x,i}(\Delta)z^{-1} + T_{x,i}(2\Delta)z^{-2} + \dots + T_{x,i}(n\Delta)z^{-n} + \dots \\ = \frac{\text{num}_0 + \text{num}_1z^{-1} + \text{num}_2z^{-2} + \dots + \text{num}_nz^{-n}}{1 + \text{den}_1z^{-1} + \text{den}_2z^{-2} + \dots + \text{den}_nz^{-n}} \cdot [I_i(0) + I_i(\Delta)z^{-1} + I_i(2\Delta)z^{-2} + \dots \\ + I_i(n\Delta)z^{-n} + \dots] \end{aligned} \quad (\text{A.7})$$

After a long manipulation of the above equation we obtain the recursive formula:

$$T_{x,i}(n\Delta) = \sum_{j=0}^n \text{num}_j I_i[(n-j)\Delta] - \sum_{j=1}^n \text{den}_j T_{x,i}[(n-j)\Delta] \quad (\text{A.8})$$

valid for the generic time $n\Delta$.

The time profile of the generic output T_x can be calculated superimposing all the partial outputs:

$$T_x(n\Delta) = \sum_{i=1}^{\text{all inputs}} T_{x,i}(n\Delta) \quad (\text{A.9})$$

Transfer function coefficients have to be calculated for each different pair of input–output e.g. air–sol temperature and the inner air temperature, inner thermal loads and the temperature of inner surface 5 and so on. All of the outputs referred to the same physical knot are later summed to obtain the global response.

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