

Certificate for [pi]_q (first 24 coefficients)

qreals certificate

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Input: the real number $x = \pi$. The continued fraction is truncated to the depth that locks the first 24 coefficients (MGO Proposition 1.1).

(a) Continued fraction and even-length MGO form

Regular continued fraction: $[3, 7, 15]$.

Even-length MGO form: $[3, 7, 14, 1]$.

The even-length form evaluates to the convergent $\frac{333}{106}$.

(b) MGO formula folded step by step

Odd positions carry $[a]_q$ with q^a above; even positions carry $[a]_{q^{-1}}$ with q^{-a} above (MGO eqn. 1.1).

innermost term a_4 = 1:

1

fold in a_3 = 14:

$$q^{14} + q^{13} + q^{12} + q^{11} + q^{10} + q^9 + q^8 + q^7 + q^6 + q^5 + q^4 + q^3 + q^2 + q + 1$$

fold in a_2 = 7: The value is the ratio $N(q)/D(q)$ with:

$$N(q) = q^{21} + 2q^{20} + 3q^{19} + 4q^{18} + 5q^{17} + 6q^{16} + 7q^{15} + 7q^{14} + 7q^{13} + 7q^{12} + \cdots + 7q^9 + 7q^8 + 7q^7 + 6q^6 + 5q^5 + 4q^4 + 3q^3 + 2q^2 + q + 1$$

(2 terms omitted)

$$D(q) = q^{21} + q^{20} + q^{19} + q^{18} + q^{17} + q^{16} + q^{15} + q^{14} + q^{13} + q^{12} + q^{11} + q^{10} + q^9 + q^8 + q^7$$

fold in a_1 = 3: The value is the ratio $N(q)/D(q)$ with:

$$N(q) = q^{24} + 2q^{23} + 4q^{22} + 7q^{21} + 10q^{20} + 13q^{19} + 16q^{18} + 19q^{17} + 21q^{16} + 22q^{15} + \cdots + 21q^9 + 20q^8 + 18q^7 + 15q^6 + 12q^5 + 9q^4 + 6q^3 + 4q^2 + 2q + 1$$

(5 terms omitted)

$$D(q) = q^{21} + 2q^{20} + 3q^{19} + 4q^{18} + 5q^{17} + 6q^{16} + 7q^{15} + 7q^{14} + 7q^{13} + 7q^{12} + \cdots + 7q^9 + 7q^8 + 7q^7 + 6q^6 + 5q^5 + 4q^4 + 3q^3 + 2q^2 + q + 1$$

(2 terms omitted)

Result. $[333/106]_q$ (the convergent, whose Taylor expansion gives $[x]_q$): The value is the ratio $N(q)/D(q)$ with:

$$N(q) = q^{24} + 2q^{23} + 4q^{22} + 7q^{21} + 10q^{20} + 13q^{19} + 16q^{18} + 19q^{17} + 21q^{16} + 22q^{15} + \cdots + 21q^9 + 20q^8 + 18q^7 + 15q^6 + 12q^5 + 9q^4 + 6q^3 + 4q^2 + 2q + 1$$

(5 terms omitted)

$$D(q) = q^{21} + 2q^{20} + 3q^{19} + 4q^{18} + 5q^{17} + 6q^{16} + 7q^{15} + 7q^{14} + 7q^{13} + 7q^{12} + \cdots + 7q^9 + 7q^8 + 7q^7 + 6q^6 + 5q^5 + 4q^4 + 3q^3 + 2q^2 + q + 1$$

(2 terms omitted)

Taylor coefficients:

$$1 + q + q^2 + q^{10} - q^{12} - q^{13} + q^{15} + q^{16} - q^{20} - 2q^{21} - q^{22} + 2q^{23} + O(q^{24})$$

(c) Independent cross-checks

- **[pass]** the first 24 coefficients are unchanged when 28 are asked for
- **[pass]** $[x+1]_q$ computed from its own continued fraction equals $q^*[x]_q + 1$
- **[na]** the series $[pi]_q$ diverges at $q=1$; $q=1$ is for rationals
- **[na]** π is irrational, so the continued fraction does not terminate

Summary: verified: truncation stable to 24, shift law $[x+1]=q[x]+1$; n/a: $q=1$ specialisation; exact rational function.

References

S. Morier-Genoud and V. Ovsienko, q -deformed rationals and q -continued fractions, Forum Math. Sigma **8** (2020), e13. Per-function theorem and check mapping: `docs/CORRECTNESS.md`.

This is a human-auditable derivation, not a formal machine proof; every line above is checkable by hand.