

Certificate for the q-integer $[5]_q$

qreals certificate

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Input: the integer $n = 5$.

(a) Continued fraction and even-length MGO form

Regular continued fraction: $[5]$.

Even-length MGO form: $[5]$.

The even-length form evaluates to the convergent 5.

(b) MGO formula folded step by step

Odd positions carry $[a]_q$ with q^a above; even positions carry $[a]_{q^{-1}}$ with q^{-a} above (MGO eqn. 1.1).

$[5]_q$ is the Gauss q-integer $1 + q + \dots + q^{n-1}$:

$$q^4 + q^3 + q^2 + q + 1$$

Result. $[5]_q$:

$$q^4 + q^3 + q^2 + q + 1$$

Taylor coefficients:

$$1 + q + q^2 + q^3 + q^4 + O(q^{12})$$

(c) Independent cross-checks

- **[pass]** $[5]_q$ at $q=1$ is 5, the ordinary value 5
- **[pass]** the Taylor expansion of the exact rational function and the truncated series agree on $q^{0..11}$: $[1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0]$

Summary: verified: $q=1$ matches 5, exact = truncated to 12.

References

S. Morier-Genoud and V. Ovsienko, q-deformed rationals and q-continued fractions, Forum Math. Sigma **8** (2020), e13. Per-function theorem and check mapping: `docs/CORRECTNESS.md`.

This is a human-auditable derivation, not a formal machine proof; every line above is checkable by hand.