

Certificate for $[\pi]_q$ (first 4 coefficients)

qreals certificate

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Input: the real number $x = \pi$. The continued fraction is truncated to the depth that locks the first 4 coefficients (MGO Proposition 1.1).

(a) Continued fraction and even-length MGO form

Regular continued fraction: $[3, 7]$.

Even-length MGO form: $[3, 7]$.

The even-length form evaluates to the convergent $\frac{22}{7}$.

(b) MGO formula folded step by step

Odd positions carry $[a]_q$ with q^a above; even positions carry $[a]_{q^{-1}}$ with q^{-a} above (MGO eqn. 1.1).
innermost term a_2 = 7:

$$\frac{q^6 + q^5 + q^4 + q^3 + q^2 + q + 1}{q^6}$$

fold in a_1 = 3:

$$\frac{q^9 + q^8 + 2q^7 + 3q^6 + 3q^5 + 3q^4 + 3q^3 + 3q^2 + 2q + 1}{q^6 + q^5 + q^4 + q^3 + q^2 + q + 1}$$

Result. $[22/7]_q$ (the convergent, whose Taylor expansion gives $[x]_q$):

$$\frac{q^9 + q^8 + 2q^7 + 3q^6 + 3q^5 + 3q^4 + 3q^3 + 3q^2 + 2q + 1}{q^6 + q^5 + q^4 + q^3 + q^2 + q + 1}$$

Taylor coefficients:

$$1 + q + q^2 + O(q^4)$$

(c) Independent cross-checks

- **[pass]** the first 4 coefficients are unchanged when 8 are asked for
- **[pass]** $[x+1]_q$ computed from its own continued fraction equals $q^*[x]_q + 1$
- **[na]** the series $[\pi]_q$ diverges at $q=1$; $q=1$ is for rationals
- **[na]** π is irrational, so the continued fraction does not terminate

Summary: verified: truncation stable to 4, shift law $[x+1]=q[x]+1$; n/a: $q=1$ specialisation; exact rational function.

References

S. Morier-Genoud and V. Ovsienko, q-deformed rationals and q-continued fractions, Forum Math. Sigma **8** (2020), e13. Per-function theorem and check mapping: `docs/CORRECTNESS.md`.

This is a human-auditable derivation, not a formal machine proof; every line above is checkable by hand.