

Certificate for $[1/2]_q$ (first 12 coefficients)

qreals certificate

May 24, 2026

Input: the real number $x = 1/2$. The continued fraction is truncated to the depth that locks the first 12 coefficients (MGO Proposition 1.1).

(a) Continued fraction and even-length MGO form

Regular continued fraction: $[0, 2]$.

Even-length MGO form: $[0, 2]$.

The even-length form evaluates to the convergent $\frac{1}{2}$.

(b) MGO formula folded step by step

Odd positions carry $[a]_q$ with q^a above; even positions carry $[a]_{q^{-1}}$ with q^{-a} above (MGO eqn. 1.1).
innermost term a_2 = 2:

$$\frac{q+1}{q}$$

fold in a_1 = 0:

$$\frac{q}{q+1}$$

Result. $[1/2]_q$ (the convergent, whose Taylor expansion gives $[x]_q$):

$$\frac{q}{q+1}$$

Taylor coefficients:

$$q - q^2 + q^3 - q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^{10} + q^{11} + O(q^{12})$$

(c) Independent cross-checks

- **[pass]** the first 12 coefficients are unchanged when 16 are asked for
- **[pass]** $[x+1]_q$ computed from its own continued fraction equals $q*[x]_q + 1$
- **[pass]** $[1/2]_q$ at $q=1$ is $1/2$, the ordinary value $1/2$
- **[pass]** the Taylor expansion of the exact rational function and the truncated series agree on $q^0..q^{11}$: $[0, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1]$

Summary: verified: truncation stable to 12, shift law $[x+1]_q = q[x]_q + 1$, $q=1$ matches $1/2$, exact = truncated to 12.

References

S. Morier-Genoud and V. Ovsienko, q-deformed rationals and q-continued fractions, Forum Math. Sigma **8** (2020), e13. Per-function theorem and check mapping: `docs/CORRECTNESS.md`.

This is a human-auditable derivation, not a formal machine proof; every line above is checkable by hand.